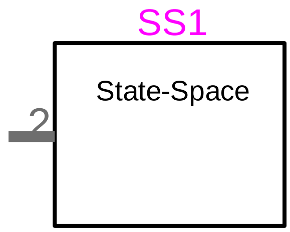
# State-space equations

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Jean Mahseredjian, 4/22/2013 7:09:00 AM

## 1 Theoretical background

This device is designed to provide the capability to use state-space equations for entering the generic relation between current and voltage of a given device. The relation may be originally available in frequency domain:

$$\tilde{\mathbf{I}} = \mathbf{Y}\tilde{\mathbf{V}} \tag{1}$$

where $\tilde{\mathbf{I}}$ is the vector of currents entering the device, $\mathbf{Y}$ is the frequency domain admittance matrix relation and $\tilde{\mathbf{V}}$ is the vector of voltages at external nodes (connectivity pins). Such an equation can be converted into an equivalent state-space representation in time-domain:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{v} \tag{2}$$

$$\mathbf{i} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{v} + \mathbf{D_1}\dot{\mathbf{v}} \tag{3}$$

This means that the size of the current vector $\mathbf{i}$ must be equal to the size of the voltage vector $\mathbf{v}$ and determines the dimensions of the square matrices $\mathbf{D}$ and $\mathbf{D_1}$. Bold characters are used to denote matrices and vectors.

EMTP can solve equations (2) and (3) for both steady-state and time-domain conditions. The steady-state (phasor) version is given by:

$$\tilde{\mathbf{X}} = \left(s\hat{\mathbf{I}} - \mathbf{A}\right)^{-1}\mathbf{B}\tilde{\mathbf{V}} \tag{4}$$

$$\tilde{\mathbf{I}} = \left[\mathbf{C}\left(s\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B} + \mathbf{D} + s\mathbf{D_1}\right]\tilde{\mathbf{V}} \tag{5}$$

where tilde is used to denote phasors and $\mathbf{I}$ is the identity matrix.

The time-domain version is based on the discretization method. When trapezoidal integration with a time-step $\Delta t$ is used, equation (2) becomes:

$$\mathbf{x}_{t+\Delta t} = \left(\mathbf{1} - \frac{\Delta t}{2}\mathbf{A}\right)^{-1}\left(\mathbf{1} + \frac{\Delta t}{2}\mathbf{A}\right)\mathbf{x}_t + \left(\mathbf{1} - \frac{\Delta t}{2}\mathbf{A}\right)^{-1}\frac{\Delta t}{2}\mathbf{B}\left(\mathbf{v}_t + \mathbf{v}_{t+\Delta t}\right) \tag{6}$$

Or in a simplified notation:

$$\mathbf{x}_{t+\Delta t} = \boldsymbol{\varphi\beta}\mathbf{x}_t + \boldsymbol{\varphi}\frac{\Delta t}{2}\mathbf{B}\mathbf{v}_t + \boldsymbol{\varphi}\frac{\Delta t}{2}\mathbf{B}\mathbf{v}_{t+\Delta t} \tag{7}$$

Equation (3) is converted into:

$$\mathbf{i}_{t+\Delta t} = \mathbf{C}\mathbf{x}_{t+\Delta t} + \mathbf{D}\mathbf{v}_{t+\Delta t} + \mathbf{i}^{c}_{t+\Delta t} \tag{8}$$

where the capacitive current $\mathbf{i}^{c}_{t+\Delta t}$ is found from:

$$\mathbf{i}^{c}_{t+\Delta t} = \frac{2}{\Delta t}\mathbf{D_1}\mathbf{v}_{t+\Delta t} - \frac{2}{\Delta t}\mathbf{D_1}\mathbf{v}_t - \mathbf{i}^{c}_t \tag{9}$$

Equations (7) and (9) are replaced into equation (8) to provide the complete formulation:

$$\mathbf{i}_{t+\Delta t} = \mathbf{C}\boldsymbol{\varphi\beta}\mathbf{x}_t + \left[\mathbf{C}\boldsymbol{\varphi}\frac{\Delta t}{2}\mathbf{B} - \frac{2}{\Delta t}\mathbf{D_1}\right]\mathbf{v}_t - \mathbf{i}^{c}_t + \left[\mathbf{C}\boldsymbol{\varphi}\frac{\Delta t}{2}\mathbf{B} + \mathbf{D} + \frac{2}{\Delta t}\mathbf{D_1}\right]\mathbf{v}_{t+\Delta t} \tag{10}$$

This system is ready for inclusion into the main system of EMTP network equations and allows achieving a simultaneous solution.

# 2 Parameters

## 2.1 Direct input method in Data tab

In the direct input method ("Use File input" is not checked), it is needed to enter state-space model matrices A, B, C, D and D1. The D1 ($\mathbf{D_1}$) matrix is optional and its text area can be left blank, in which case it will become zero.

Matrices are entered line-by-line in free format style. Brackets ("[" for opening and "]" for closing) are optional and will be automatically discarded. It is also acceptable to use semicolons ";" for separating matrix lines entered on the same text line.

The dimensions of the square matrix A define the number of states n_states.

Matrix B must have n_states rows and n_inputs (size of voltage vector and the number of connectivity pins) columns.

Matrix C must have n_inputs rows and n_states columns.

Matrix D must have n_inputs rows and n_inputs columns.

Matrix D1 is optional and must have n_inputs rows and n_inputs columns.

All matrix dimensions are tested when entered directly. Data input errors are detected in EMTP when named values are used.

## 2.2 File input method in Data tab

In the File input method ("Use File input" is checked) the user must enter the name of the file which contains all necessary matrix data.

The format of the file is as follows:

- First line, free format, space separated numbers: n_outs n_inputs n_states n_D1

  In this version the number of outputs n_outs is strictly equal to the number of inputs n_inputs. This means that the size of the current vector is equal to the size of the voltage vector. The size of matrix D1 is given by n_D1, it is equal to 0 when there is no D1 matrix or n_inputs otherwise.
- Matrix A rows, free format, space separated cells, one matrix row by text line
- Matrix B rows, free format, space separated cells, one matrix row by text line
- Matrix C rows, free format, space separated cells, one matrix row by text line
- Matrix D rows, free format, space separated cells, one matrix row by text line
- Optional matrix D1 rows, free format, space separated cells, one matrix row by text line

## 2.3 IC data tab: Initial conditions for states

This tab allows entering initial values for states: the History matrix. The initial value can be any real number. The History matrix is a two column matrix, entered line-by-line and in free format. First column is for identifying initialized state number, second column is for initial value of state. If a state number is omitted it is automatically initialized to 0. The History matrix can be left empty when there are no initial conditions for all states.

# 3 Device pins

When the device data is completed its bundle pin is automatically updated to include the required number of pins. This is the size of the voltage vector. In the figure below the number 2 indicates two pins. To connect to any pin it is necessary to extend the bundle signal and right-click to select the Breakout command. The available pins are listed in the appearing panel. Non-required pins must be erased from this panel before clicking the OK button. In the case of Figure 2 all pins have been selected.

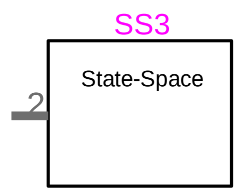


**Figure 1 A state-space device with two pins**

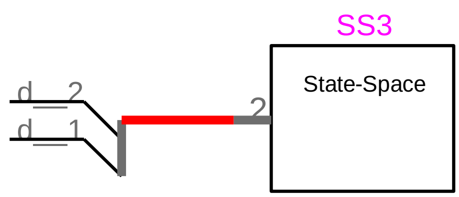


**Figure 2 Selected connectivity pins after using the Breakout command**

The pin names are standard, so that several devices can be connected together using the bundle signal. In the case of Figure 3 two state-space devices are connected in parallel. Special connections with bundle pins can be achieved using the “Node connecter” device.

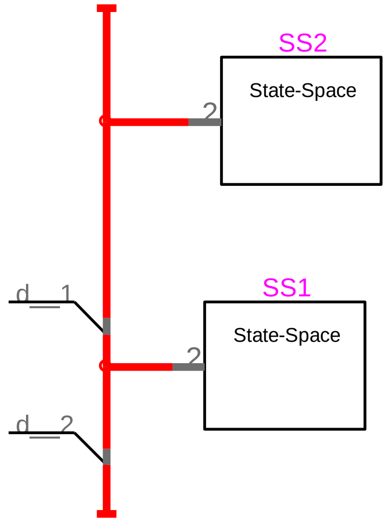


**Figure 3 State-space devices connected in parallel**

# 4 Netlist format

This device allows method-based scripting. The object data and methods are described in the script file referenced by the device Script.Open.Dev attribute.

Sample listing of Netlist data:

```
_ss;SS1;2;2;s1d__1,s1d__2,
2,2,1,0,0,1,
test_matrix.dat,
_ss;SS2;2;2;s1d__1,s1d__2,
2,2,1,0,2,,
-3.333333333333333e+003
3.333333333333333e+003 -3.333333333333333e+003
-0.333333333333333
0.333333333333333
0.833333333333333 -0.833333333333333
-0.833333333333333 +0.833333333333333
100e-06 -100e-06
-100e-06 100e-06
```

| Field | Description |
|---|---|
| _ss | Part name |
| SS1 | Instance name, any name. |
| 2 | Total number of pins |
| 2 | Number of pins given in this data section |
| s1d__1 | First pin in bundle s1 |
| s1d__2 | Second pin in bundle s1 |
| 2 | Number of outputs (size of current vector) |
| 2 | Number of inputs (size of voltage vector) |
| 1 | Number of states |
| 0 | Number of rows in the history matrix |
| 0 | Number of rows in D1 matrix |

| | |
|---|---|
| 1 | 1 means file input method |
| file name | File name for file input method or matrices |
| Matrices | Matrices A, B, C, D, D1 and History. D1 and History are optional. All matrices are appearing row-by-row. The History data (as many rows as needed) is entered as follows: state_number,history, |

Device data fields are saved in ParamsA and ModelData device attributes.

# 5 Steady-state model

The steady-state model is given by equation (5).

# 6 Initial conditions

Automatic initial conditions are found from the steady-state solution. Manual initial conditions can be provided for selected state variables.

# 7 Frequency Scan model

Similar to the steady-state. The branch impedance is found at each frequency.

# 8 Time-domain model

The device is discretized according to the integration time-step and solved at each simulation time-point. See equation (10) for trapezoidal integration.

# 9 Example

This simple example compares the simulation of an actual circuit to its equivalent state-space model. The circuit drawing is shown in Figure 3. The design file is test_ss.ecf.
It has one independent state variable and two external connectivity pins (nodes). The current and voltage vectors have a size of 2. The currents i1 and i2 are entering the circuit pins as shown on the diagram.

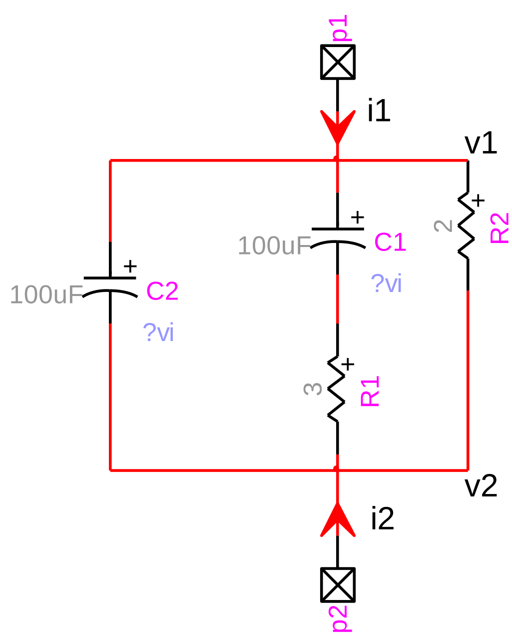


**Figure 4 Equivalent circuit drawing**

The symbolic matrix A is given by:

$$\mathbf{A} = -\frac{1}{R_1 C_1} \tag{11}$$

The other matrices are:

$$\mathbf{B} = \begin{bmatrix} \frac{1}{R_1 C_1} & -\frac{1}{R_1 C_1} \end{bmatrix} \tag{12}$$

$$\mathbf{C} = \begin{bmatrix} \frac{-1}{R_1} \\ \frac{1}{R_1} \end{bmatrix} \tag{13}$$

$$\mathbf{D} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} - \frac{1}{R_2} \\ -\frac{1}{R_1} - \frac{1}{R_2} & \frac{1}{R_1} + \frac{1}{R_2} \end{bmatrix} \tag{14}$$

$$\mathbf{D_1} = \begin{bmatrix} C_2 & -C_2 \\ -C_2 & C_2 \end{bmatrix} \tag{15}$$

Since both state-space and circuit representations are mathematically identical, the simulation waveforms from both circuits are the same.