## Input Impedance


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## 1 Description

This device works with 1-phase and 3-phase signals. It is used to calculate the input impedance seen between the connected node and ground or optionally between two nodes.

The input impedance is calculated for the selected range of Frequency scan. The Frequency scan data is selected and entered through Simulation Options.

All independent voltage and current sources are killed during the frequency scan. Machine models are replaced by an equivalent impedance to ground. A killed voltage source is a short-circuit and a killed current source is an open-circuit.

### 1.1 Parameters and Rules

In the 1-phase version it is only allowed to select an extra pin for computing the input impedance between two nodes instead of one node and the ground node. The computation is performed by injecting a 1 Ampere current source in the shown direction and computing the voltage between the positive node (injection point, arrow) and negative node (ground or given node).

In the 3-phase version, in addition to the selection of an extra pin, it is needed to select the type of input impedance calculation. The available options are positive-sequence, negative-sequence and zero-sequence. The computation method is explained in the section below.

If more than one device of this type is used in a given design, then the simulation results will be correct only if the related circuits are decoupled.
The input impedance waveform will appear in the scope group Input Impedance (zin). It is identified by the device name.

### 1.2 Computation method in the 3-phase case

When the user selects the 3-phase version of this device by changing attached signal Line Type, this device is internally replaced by three independent current sources of 1 A . The current source phases are related to the choice of input impedance computation: zero-sequence, positive-sequence and negative-sequence.

If index 0 is used for zero-sequence, index 1 is used for positive-sequence and index 2 for negative-sequence then the injected phase currents (using phasor notation) for each sequence selection are given by:

$$
\begin{align*}
& {\left[\begin{array}{l}
\mathrm{I}_{\mathrm{oa}} \\
\mathrm{I}_{\mathrm{ob}} \\
\mathrm{I}_{\mathrm{oc}}
\end{array}\right]=\left[\begin{array}{lll}
1 & & \\
& 1 & \\
& & 1
\end{array}\right]}  \tag{1}\\
& {\left[\begin{array}{l}
\mathrm{I}_{0} \\
\mathrm{I}_{1 \mathrm{~b}} \\
\mathrm{I}_{\mathrm{cc}}
\end{array}\right]=\left[\begin{array}{lll}
1 & & \\
& \mathrm{a}^{2} & \\
& & \mathrm{a}
\end{array}\right]}  \tag{2}\\
& {\left[\begin{array}{l}
\mathrm{I}_{\mathrm{Ia}} \\
\mathrm{I}_{2 \mathrm{~b}} \\
\mathrm{I}_{2 \mathrm{c}}
\end{array}\right]=\left[\begin{array}{lll}
1 & & \\
& \mathrm{a} & \\
& & \mathrm{a}^{2}
\end{array}\right]} \tag{3}
\end{align*}
$$

The operator $a$ is given by:

$$
\begin{equation*}
a=1 \underline{120^{\circ}} \tag{4}
\end{equation*}
$$

For each current $I_{0}, I_{1}$ and $I_{2}$, the phasor amplitude is 1 and the phase angle is 0 .
In the general case when looking into three nodes $a, b$ and $c$, the input impedance is related to the injected currents as follows (bold characters are used to denote vectors and matrices):

$$
\begin{gather*}
\mathbf{V}_{\mathrm{abc}}=\mathrm{Z}_{\mathrm{abc}}  \tag{5}\\
{\left[\begin{array}{c}
\mathrm{V}_{\mathrm{a}} \\
\mathrm{~V}_{\mathrm{b}} \\
\mathrm{~V}_{\mathrm{c}}
\end{array}\right]=\left[\begin{array}{lll}
\mathrm{Z}_{\mathrm{aa}} & \mathrm{Z}_{\mathrm{ab}} & \mathrm{Z}_{\mathrm{ac}} \\
\mathrm{Z}_{\mathrm{ba}} & \mathrm{Z}_{\mathrm{bb}} & \mathrm{Z}_{\mathrm{bc}} \\
\mathrm{Z}_{\mathrm{ca}} & \mathrm{Z}_{\mathrm{cb}} & \mathrm{Z}_{\mathrm{cc}}
\end{array}\right]\left[\begin{array}{c}
\mathrm{I}_{\mathrm{a}} \\
\mathrm{I}_{\mathrm{b}} \\
\mathrm{I}_{\mathrm{c}}
\end{array}\right]} \tag{6}
\end{gather*}
$$

If the power-variant Fortescue transformation is used the transformation matrix is given by:

$$
A=\left[\begin{array}{ccc}
1 & 1 & 1  \tag{7}\\
1 & a^{2} & a \\
1 & a & a^{2}
\end{array}\right]
$$

with

$$
A^{-1}=\frac{1}{3}\left[\begin{array}{ccc}
1 & 1 & 1  \tag{8}\\
1 & a & a^{2} \\
1 & a^{2} & a
\end{array}\right]
$$

It is noticed that:

$$
\begin{equation*}
\mathbf{A}^{\mathrm{t}} \mathbf{A}^{*}=\mathbf{3} \tag{9}
\end{equation*}
$$

where the superscript " $t$ " means transposition and "*" means complex-conjugate. This transformation matrix is applied on phase quantities to result into:

$$
\begin{gather*}
\mathrm{V}_{\mathrm{abc}}=\mathbf{A} \mathrm{V}_{\mathbf{0 1 2}}  \tag{10}\\
{\left[\begin{array}{l}
\mathrm{V}_{\mathrm{a}} \\
\mathrm{~V}_{\mathrm{b}} \\
\mathrm{~V}_{\mathrm{c}}
\end{array}\right]=\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & \mathrm{a}^{2} & \mathrm{a} \\
1 & \mathrm{a} & \mathrm{a}^{2}
\end{array}\right]\left[\begin{array}{l}
\mathrm{V}_{0} \\
\mathrm{~V}_{1} \\
\mathrm{~V}_{2}
\end{array}\right]}  \tag{11}\\
\mathbf{I}_{\mathrm{abc}}=\mathbf{A l}_{\mathbf{0 1 2}} \tag{12}
\end{gather*}
$$

$$
\left[\begin{array}{l}
I_{a}  \tag{13}\\
I_{b} \\
I_{c}
\end{array}\right]=\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a^{2} & a \\
1 & a & a^{2}
\end{array}\right]\left[\begin{array}{l}
I_{0} \\
I_{1} \\
I_{2}
\end{array}\right]
$$

When the above equations are replaced into equation (6), it becomes:

$$
\begin{equation*}
A V_{012}=Z A I_{012} \tag{14}
\end{equation*}
$$

which is equivalent to:

$$
\begin{equation*}
V_{012}=A^{-1} Z A I_{012} \tag{15}
\end{equation*}
$$

and the sequence impedance matrix is defined as:

$$
\begin{equation*}
Z_{012}=A^{-1} Z A \tag{16}
\end{equation*}
$$

The generic version of this impedance is given by:
with:

$$
\begin{gather*}
\mathrm{Z}_{\mathrm{s} 0}=\frac{1}{3}\left[\mathrm{Z}_{\mathrm{aa}}+\mathrm{Z}_{\mathrm{bb}}+\mathrm{Z}_{\mathrm{cc}}\right]  \tag{18}\\
\mathrm{Z}_{\mathrm{S} 1}=\frac{1}{3}\left[\mathrm{Z}_{\mathrm{aa}}+\mathrm{aZ} \mathrm{Z}_{\mathrm{bb}}+\mathrm{a}^{2} \mathrm{Z}_{\mathrm{cc}}\right]  \tag{19}\\
\mathrm{Z}_{\mathrm{S} 2}=\frac{1}{3}\left[\mathrm{Z}_{\mathrm{aa}}+\mathrm{a}^{2} \mathrm{Z}_{\mathrm{bb}}+\mathrm{a} \mathrm{Z}_{\mathrm{cc}}\right]  \tag{20}\\
\mathrm{Z}_{\mathrm{M} 0}=\frac{1}{3}\left[\mathrm{Z}_{\mathrm{bc}}+\mathrm{Z}_{\mathrm{ca}}+\mathrm{Z}_{\mathrm{ab}}\right]  \tag{21}\\
\mathrm{Z}_{\mathrm{M} 1}=\frac{1}{3}\left[\mathrm{Z}_{\mathrm{bc}}+\mathrm{aZ}_{\mathrm{ca}}+\mathrm{a}^{2} \mathrm{Z}_{\mathrm{ab}}\right]  \tag{22}\\
\mathrm{Z}_{\mathrm{M} 2}=\frac{1}{3}\left[\mathrm{Z}_{\mathrm{bc}}+\mathrm{a}^{2} \mathrm{Z}_{\mathrm{ca}}+\mathrm{a} \mathrm{Z}_{\mathrm{ab}}\right] \tag{23}
\end{gather*}
$$

The relation between a given sequence voltage and current is the diagonal element in the matrix of equation (17), but under generic conditions coupling between sequence impedances does not create a diagonal matrix and a given sequence current contributes to all sequence voltages.
If the network is balanced $\left(Z_{a c}=Z_{b c}=Z_{a b}\right.$ and $\left.Z_{a a}=Z_{b b}=Z_{c c}\right)$ then its symbolic impedance matrix in phasedomain is given by:

$$
Z_{a b c}=\left[\begin{array}{lll}
Z_{S} & Z_{M} & Z_{M}  \tag{24}\\
Z_{M} & Z_{S} & Z_{M} \\
Z_{M} & Z_{M} & Z_{S}
\end{array}\right]
$$

According to equations (17) to (23), the sequence impedance matrix now becomes:

$$
Z_{012}=\left[\begin{array}{ccc}
Z_{S}+2 Z_{M} & 0 & 0  \tag{25}\\
0 & Z_{S}-Z_{M} & 0 \\
0 & 0 & Z_{S}-Z_{M}
\end{array}\right]
$$

There is no more coupling between sequence networks and three independent circuits are created. This is conformal to the actual definition of sequence impedances.
Since the observed network is not always balanced, EMTP-RV uses equation (14) to calculate the phase voltages according to the vector $\mathrm{I}_{012}$ with one non-zero element:

$$
\begin{equation*}
\mathrm{V}_{\mathrm{abc}}=\mathrm{ZA} \mathrm{I}_{012} \tag{26}
\end{equation*}
$$

Then the sequence impedances are calculated using equation (10):

$$
\begin{equation*}
V_{012}=A^{-1} V_{a b c} \tag{27}
\end{equation*}
$$

When the network is balanced, the resulting impedances are those of equation (25). When the network is not balanced then the selected input sequence impedance becomes the diagonal term of equation (17).

In the previous (before version 2.1) versions of EMTP-RV, the sequence impedances were calculated by simply measuring the voltage on phase a node(s) at the injection location. Since in a balanced network the application of equation (27) results into $V_{a}$ for each sequence, this is the same as equation (25). In the unbalanced case, however, the answer was different from the application of equation (27). This is a matter of definition, but the user can still access this computation by measuring directly (voltage scope) the voltage on phase a at the connection point of the Input Impedance device.

## 2 Netlist format

This device allows method-based scripting. The object data and methods are described in the script file referenced by the device Script.Open.Dev attribute.

A Netlist example for the default 1-phase case is given below:

```
Zin;ZIN1;1;1;s1,
1,?z,
```

In the 3-phase case when the device has been given a 3-phase signal, the Netlist becomes:

```
_Zin;ZIN2a;1;1;s2a,
1,?z,
Zin;ZIN2b;1;1;s2b,
Zin;ZIN2c;1;1;s2c,
```

This will result into the default selection of positive sequence. There are now three devices to indicate the 3phase version.
If the device has been given a 3-phase signal and reopened and saved, then the Netlist becomes:

```
Zin;ZIN2a;1;1;s2a,
```

3,2,?z,
_Zin;ZIN2b;1;1;s2b,
_Zin;ZIN2c;1;1;s2c,

The selection is now negative sequence.
Data is saved only into the ParamsA attribute. The ParamsB and ParamsC attributes are empty. In the 3phase version EMTPWorks automatically generates 3 separate devices, one per phase. The phase identification character ( $a, b$ or $c$ ) is automatically appended to the device instance name and signals.

The following Table describes data for the 3-phase version shown above.

| Field | Description |
| :--- | :--- |
| Zin | Part name |
| ZIN1 | Instance name, any name |
| 1 | Total number of pins |
| 1 | Number of pins given in this data section |
| s2a | Signal name connected to k-pin, injection point, any name |
| mm | Signal name connected to m-pin, optional |
| 3 | ParamsA section, mean 3-phase, 1 means 1-phase. The 3-phase version is <br> automatically detected when this value is 1, but 3 devices are available. |
| 2 | Appears only in the 3-phase case, optional, gives the sequence selection, 0, 1, 2. |
| $? \mathrm{z}$ | Always needed scope reques. |

## 3 Load-Flow model

This device becomes an open-circuit in the Load-Flow solution.

## 4 Steady-state model

This device becomes an open-circuit in the Steady-State solution.

## 5 Frequency Scan model

This device becomes a current source of 1 A injected at selected frequencies.

## 6 Time-domain model

This device becomes an open-circuit in the time-domain solution.

